OCR
Oxford Cambridge and RSA

Monday 05 October 2020 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Find the mean value of $\mathrm{f}(x) = x^2 + 6x$ over the interval $[0, 3]$. **[2]**

2 Find an expression for $1\times 2^2 + 2\times 3^2 + 3\times 4^2 + \ldots + n(n+1)^2$ in terms of n. Give your answer in fully factorised form. **[3]**

3 You are given the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$.

(a) Find $\mathbf{A}^4$. **[1]**

(b) Describe the transformation that **A** represents. **[2]**

The matrix **B** represents a reflection in the plane $x = 0$.

(c) Write down the matrix **B**. **[1]**

The point P has coordinates (2, 3, 4). The point P' is the image of P under the transformation represented by **B**.

(d) Find the coordinates of P'. **[1]**

4 **In this question you must show detailed reasoning.**

(a) Determine the square roots of 25i in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $0 \leqslant \theta < 2\pi$. **[3]**

(b) Illustrate the number 25i and its square roots on an Argand diagram. **[1]**

5 By expanding $\left(z^2 + \frac{1}{z^2}\right)^3$, where $z = \mathrm{e}^{\mathrm{i}\theta}$, show that $4\cos^3 2\theta = \cos 6\theta + 3\cos 2\theta$. **[5]**

6 The equations of two non-intersecting lines, l_1 and l_2, are

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \qquad l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}.$$

Find the shortest distance between lines l_1 and l_2. **[5]**

7 Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. **[5]**

8 **(a)** Using exponentials, show that $\cosh 2u \equiv 2\sinh^2 u + 1$. **[2]**

(b) By differentiating both sides of the identity in part **(a)** with respect to u, show that

$\sinh 2u \equiv 2\sinh u \cosh u$. **[1]**

(c) Use the substitution $x = \sinh^2 u$ to find $\int \sqrt{\frac{x}{x+1}}\,dx$. Give your answer in the form $a\sinh^{-1} b\sqrt{x} + f(x)$ where a and b are integers and $f(x)$ is a function to be determined. **[5]**

(d) Hence determine the exact area of the region between the curve $y = \sqrt{\frac{x}{x+1}}$, the x-axis, the line $x = 1$ and the line $x = 2$. Give your answer in the form $p + q\ln r$ where p, q and r are numbers to be determined. **[2]**

9 You are given that the cubic equation $2x^3 + px^2 + qx - 3 = 0$, where p and q are real numbers, has a complex root $\alpha = 1 + i\sqrt{2}$.

(a) Write down a second complex root, β. **[1]**

(b) Determine the third root, γ. **[2]**

(c) Find the value of p and the value of q. **[2]**

(d) Show that if n is an integer then $\alpha^n + \beta^n + \gamma^n = 2 \times 3^{\frac{1}{2}n} \times \cos n\theta + \frac{1}{2^n}$ where $\tan\theta = \sqrt{2}$. **[4]**

10 A particle of mass 0.5 kg is initially at point O. It moves from rest along the x-axis under the influence of two forces F_1 N and F_2 N which act parallel to the x-axis. At time t seconds the velocity of the particle is $v\,\text{m s}^{-1}$.
F_1 is acting in the direction of motion of the particle and F_2 is resisting motion.

In an initial model

- F_1 is proportional to t with constant of proportionality $\lambda > 0$,
- F_2 is proportional to v with constant of proportionality $\mu > 0$.

(a) Show that the motion of the particle can be modelled by the following differential equation.

$$\frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t} = \lambda t - \mu v$$ **[2]**

(b) Solve the differential equation in part **(a)**, giving the particular solution for v in terms of t, λ and μ. **[7]**

You are now given that $\lambda = 2$ and $\mu = 1$.

(c) Find a formula for an approximation for v in terms of t when t is large. **[2]**

In a refined model

- F_1 is constant, acting in the direction of motion with magnitude 2 N,
- F_2 is as before with $\mu = 1$.

(d) Write down a differential equation for the refined model. **[1]**

(e) Without solving the differential equation in part **(d)**, write down what will happen to the velocity in the long term according to this refined model. **[1]**

11 A curve has cartesian equation $x^3 + y^3 = 2xy$.

C is the portion of the curve for which $x \geqslant 0$ and $y \geqslant 0$. The equation of C in polar form is given by $r = \mathrm{f}(\theta)$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Find $\mathrm{f}(\theta)$. **[2]**

(b) Find an expression for $\mathrm{f}\left(\frac{1}{2}\pi - \theta\right)$, giving your answer in terms of $\sin\theta$ and $\cos\theta$. **[2]**

(c) Hence find the line of symmetry of C. **[1]**

(d) Find the value of r when $\theta = \frac{1}{4}\pi$. **[1]**

(e) By finding values of θ when $r = 0$, show that C has a loop. **[2]**

12 Show that $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1 - x^4}\,\mathrm{d}x = \ln\left(a + \sqrt{b}\right) + \frac{\pi}{c}$ where a, b and c are integers to be determined. **[6]**

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